Manifestation of Quantum Interference in Lasing Without Inversion*

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Z. Naturforsch. **54a**, 33–38 (1999); received May 5, 1998

In terms of quantum interference we demonstrate the physical mechanisms which lead to light amplification without population inversion. The similarities and differences between the two model schemes, namely, Λ and V-type, are emphasized. A coherent radiation field, on one hand, which drives one of the lasing levels, yields the quantum mechanical two paths via Autler-Townes splittings. On the other hand, the spontaneous emission in this driving transition plays a key role in the asymmetries between the absorption and the stimulated emission in the lasing transition.

I. Introduction

It was first pointed out by Javan [1] that certain effects in a three-level system cannot be predicted by considering the population difference alone. The possibility of lasers without population inversion was noted many years ago [2]. It was, however, away from the central attention at that time and did not lead to any attempt of experimental realization. The current idea of lasing without inversion (LWI) was proposed in the late 80's independently by several authors [3–5] and has been received considerable attention [6]. Various theoretical models have been examined [7], and experimental observations have also been achieved [8] for inversionless amplification or lasing.

Particular attention has been paid to two among the many models of active mediums, namely Λ-type and V-type (Figure 2). However, as is pointed out by Grynberg, Pinard, and Mandel [9], the relations between these effects in various models of an atomic system have seldom been clarified. Since they are not identical, what can be the basic similarities and differences in the LWI mechanisms between these two models? Describing the physical mechanism in terms of quantum interference needs quantum mechanical two paths and it is provided by the coherent driving field via Autler-Townes doublet [10]. What, then, determines the patterns of the interference?

* It is a pleasure to dedicate this paper to Prof. Georg Süssmann; excellent physicist and friend, our teacher in so many fields of thought.

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One of the first proposals of LWI described a system where the two upper levels decay to the same continuum, as depicted in Figure 1 (a). In such a system the equations of motion for the probability amplitudes are given as

$$\dot{a}_1(t) = -(\gamma_1 + i\Delta_1)\cdot a_1(t) - \frac{\sqrt{\gamma_1\gamma_2}}{2}\, a_2(t) - i\Omega_1\, b(t)\,, \tag{1a}$$

$$\dot{a}_2(t) = -(\gamma_2 + i\Delta_2)\cdot a_2(t) - \frac{\sqrt{\gamma_1\gamma_2}}{2}\, a_1(t) - i\Omega_2\, b(t)\,, \tag{1b}$$

where γ_i is the decay rate of the state $|a_i\rangle$, Δ_i and Ω_i are the detuning and Rabi frequency of the probe field. When the two upper levels decay to the common continuum (where we have $\sqrt{\gamma_1\gamma_2}$ terms), it leads to the cancellation of absorption [4]. The upper level coherence is built via the Fano-type interference [11–13].

One the other hand, LWI can be achieved in a three-level scheme with ground state doublet [5, 14], as in Fig. 2b, where the quantum coherence in the doublet can lead to absorption cancellation. In this case the probability of absorption of the probe field is given by the mod-

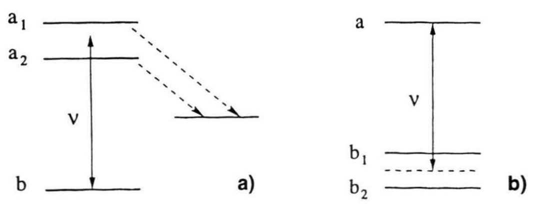

Fig. 1. a) Two upper levels decay to a common continuum [4], b) two lower levels are coherently prepared [5, 14].

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ulus square of each contribution as

$$\begin{aligned} P_{\text{abs}} &= C|\Omega_1^2 b_1(0) + \Omega_2^2 b_2(0)|^2 \\ &= C(\Omega_1^2 \rho_{b1b1} + \Omega_1 \Omega_2 \rho_{b1b2} \\ &\quad + \Omega_2 \Omega_1 \rho_{b2b1} + \Omega_2^2 \rho_{b2b2}) , \end{aligned} \tag{2}$$

where C is an overall constant. We note that the off diagonal elements ρ_{b1b2} and ρ_{b1b2} can cancel the terms ρ_{b1b1} and ρ_{b2b2}. This *quantum beat*-type interference results from the atomic coherence.

It is, however, hard to find the atomic system which can do the job (with closely spaced upper-level doublet or lower-level doublet). Nevertheless one can establish such systems by using the dressed states. Applying a coherent radiation field to drive one of the levels in the lasing transition, one can have "designer atom" or "man-made Fano interference". The scheme of utilizing Fano interference may be established by driving the upper level with a coherent field like the Λ-type configuration, see Figure 2a. Likewise, the latter case of utilizing the ground-state coherence may be achieved by driving the lower level with a coherent field like V-type configuration, see Figure 2b.

For a Λ-type system [15], if we assume a weak probe field, the off-diagonal element of the density matrix ρ_{ab} which governs the polarization for the lasing transition is written as

$$\rho_{ab}^{\Lambda} = \frac{i\Omega_p}{\Gamma_{ab}\Gamma_{cb} + \Omega^2} \cdot \left[\Gamma_{cb}(\rho_{aa}^{(0)} - \rho_{bb}^{(0)}) + \frac{\Omega^2}{\Gamma_{ca}}(\rho_{cc}^{(0)} - \rho_{aa}^{(0)}) \right], \tag{3}$$

where Ω_p and Ω are the Rabi frequencies of the probe field and the driving field, respectively. $\rho_{\alpha\alpha}$'s are the zeroth order (in the probe field) population in each level $|\alpha\rangle$. $\Gamma_{\alpha\beta} = \gamma_{\alpha\beta} + i\Delta_{\alpha\beta}$, where $\gamma_{\alpha\beta}$'s are the decay rates of the coherence $\rho_{\alpha\beta}$, and $\Delta_{\alpha\beta}$'s are the differences in detunings. Here we can see that, even if the populations are not inverted ($\rho_{aa}^{(0)} < \rho_{bb}^{(0)}$), amplification is possible by the term with $\rho_{cc}^{(0)} - \rho_{aa}^{(0)}$ which comes from the lower-level atomic coherence ρ_{cb}.

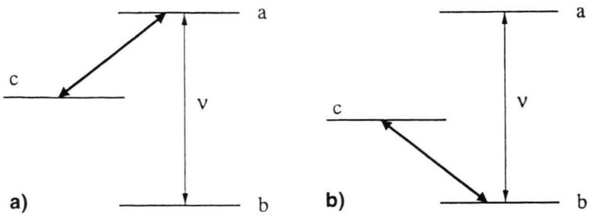

Fig. 2. 3-level systems. Lasing fields are coupling to $|a\rangle$ to $|b\rangle$. (a) Λ scheme. (b) V scheme.

Whereas, for a V-type system, it is given by

$$\rho_{ab}^{V} = \frac{i\Omega_p}{\Gamma_{ab}\Gamma_{ac} + \Omega^2} \cdot \left[\Gamma_{ac}(\rho_{aa}^{(0)} - \rho_{bb}^{(0)}) + \frac{\Omega^2}{\Gamma_{bc}}(\rho_{bb}^{(0)} - \rho_{cc}^{(0)}) \right]. \tag{4}$$

We can also see the possibility of LWI due to the term corresponding to $\rho_{bb}^{(0)} - \rho_{cc}^{(0)}$ which comes from the upper-level atomic coherence ρ_{ac}.

Since the Λ system is to utilize the Fano interference, whereas the V system is to achieve the quantum beat-type interference, they have different origins of gain without inversion. However, as we write the expression of ρ_{ab}^{Λ} and ρ_{ab}^{V}, there comes one question: Why do they look so much alike when the physics seems to be so different? In this paper we examine this question and clarify the similarities and differences in physical mechanisms of Λ and V-type LWI.

II. Λ-Type System

Let us first consider the Λ-type system. The dressed states given by the interaction between the atom and the driving field can be written as (see Fig. 3)

$$\begin{aligned} |+,n\rangle &= \frac{1}{\sqrt{2}}(|a,n\rangle + |c,n+1\rangle) , \\ |-,n\rangle &= \frac{1}{\sqrt{2}}(|a,n\rangle - |c,n+1\rangle) \end{aligned} \tag{5}$$

with eigenvalues $\pm\Omega \approx \pm g\sqrt{n+1}$, where n is the number of photon in the driving field mode and g is the coupling constant between the levels $|a\rangle$ and $|c\rangle$. We have assumed the resonance condition for the driving field.

We then can write the state vector in terms of these dressed states ($|A_{1,2}\rangle \equiv |\pm, n\rangle$, ($|D_{1,2}\rangle \equiv |\pm, n-1\rangle$) such as

$$\begin{aligned} |\Psi(t) = [&a_1(t)|A_1\rangle + a_2(t)|A_2\rangle + b(t)|B\rangle]|\{0\}\rangle \\ &+ \sum_k d_k(t)|D_1\rangle|1_k\rangle + \sum_q d_q(t)|D_2\rangle|1_q\rangle \end{aligned} \tag{6}$$

The equations of motion for the probability amplitudes are given by

$$\dot{a}_1(t) = -\Gamma_1 a_1(t) - f a_2(t) - i\Omega_1 b(t) , \tag{7a}$$

$$\dot{a}_2(t) = -\Gamma_2 a_2(t) - f a_1(t) - i\Omega_2 b(t) , \tag{7b}$$

$$\dot{b}(t) = -i\Omega_1 a_1(t) - i\Omega_2 a_2(t) , \tag{7c}$$

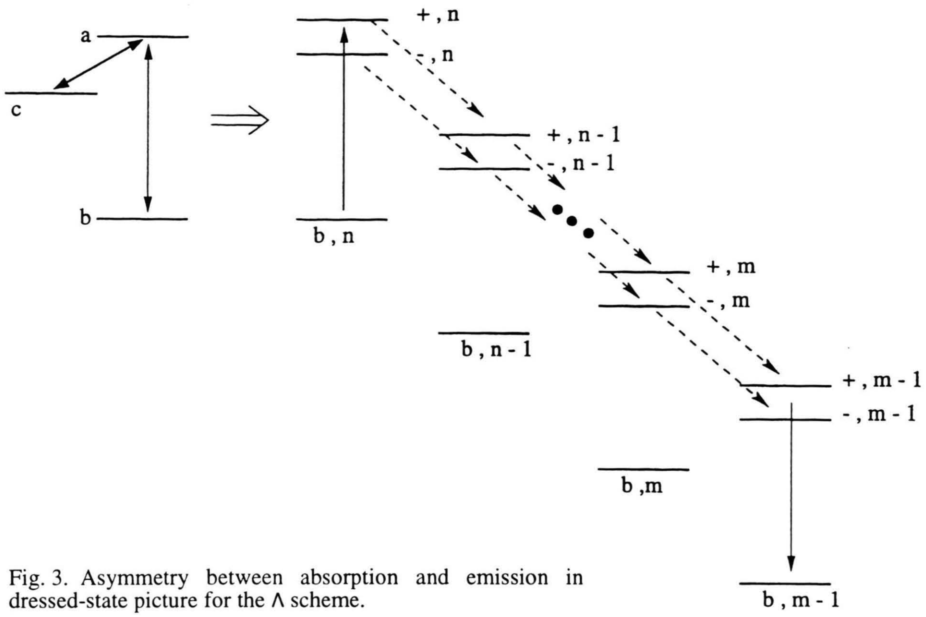

Fig. 3. Asymmetry between absorption and emission in dressed-state picture for the Λ scheme.

where

$$\Gamma_j=\frac{\gamma_j+\gamma_j'}{2}+i\Delta_j\,,\quad f=\frac{\sqrt{\gamma_1\gamma_2}}{2}+\frac{\sqrt{\gamma_1'\gamma_2'}}{2}\,,\tag{8}$$

$\gamma_i(\gamma_i')$ is the decay rate from level $|A_i\rangle$ to $|D_1\rangle$ ($|D_2\rangle$), and $\Omega_{1,2}=\Omega_p/\sqrt{2}$ for the Rabi frequency of the probe field Ω_p. The probe field is also assumed to be on resonance ($\nu=\omega_{ab}$), for simplicity, which gives $\Delta_{1,2}=\pm\Omega$.

For a weak probe field, following [4], the transition rate for absorption can be given by

$$W_{ab}=\frac{\partial}{\partial t}|b(t)|^2\,,\tag{9}$$

where we neglect the time derivatives in (7a, 7b) by assuming large Γ_i's compared to the derivatives. Then, with $b(t)\cong 1$ we have

$$a_1(t)=\frac{i}{\Gamma_1\Gamma_2-f^2}[\Omega_2 f-\Omega_1\Gamma_2]\,,$$
$$a_2(t)=\frac{i}{\Gamma_1\Gamma_2-f^2}[\Omega_1 f-\Omega_2\Gamma_1]\,.\tag{10}$$

By substituting (10) in (7c) we have the transition rate for absorption as

$$W_{ab}=0\,.\tag{11}$$

We now compare this result with a situation where there is no interference, i.e. where there is no cross coupling term ($f=0$) in (7). In the same manners as we did before, with $f=0$ we have

$$\dot{b}(t)=-\left[\frac{\Omega_1^2}{\Gamma_1}+\frac{\Omega_2^2}{\Gamma_2}\right]\tag{12}$$

and therefore

$$W_{ab(f=0)}=\frac{\Omega_p^2\gamma}{(\gamma^2/16)+\Omega^2}\,\frac{1}{2}\,.\tag{13}$$

Comparing (11) with (13), we can see the quantum interference involved in the absorption for the Λ scheme is destructive. If we consider an upper-level–driven cascade scheme, we note that the only difference is the direction of the spontaneous decay in the driving channel. This difference, however, leads to enhancement of absorption for that type of cascade scheme.

On the other hand, the transition rate for stimulated emission can be given by

$$W_e=|b(t=\infty)|^2/\tau\,,\tag{14}$$

where $b(t=0)=0$ and $\tau=\int_0^\infty dt|a_1(t)|^2+|a_2(t)|^2$. Here we obtain $a_j(t)$ by taking $\Omega_j=0$ in (7a, 7b) with given initial conditions. For the emission process, the initial state is a combination of the two dressed states (see Figure 3).

It is emphasized in quantum jump approach [16] as 'A coherent evolution begins and ends by quantum jumps'. In this context the spontaneous emission in the driving transition yields the initial state for the emission process.

Taking the initial state as $|c,n\rangle$, with $a_1(0)=-a_2(0)=1/\sqrt{2}$ we find the emission rate as

$$W_e=\frac{\Omega_p^2\gamma}{\Omega^2}\left[\frac{2\Omega^2}{(\gamma^2/2)+4\Omega^2}\right]. \tag{15}$$

Let us now examine the emission rate where the atoms are initially a complete incoherent superposition of the two lower levels. Then, the emission rate can be obtained as

$$W_{e(\mathrm{incoh})}=\frac{\Omega_p^2\gamma}{(\gamma^2/16)+\Omega^2}\,\frac{1}{4}. \tag{16}$$

Comparing (15) and (16), the quantum interference involved in the emission for the Λ scheme shows constructive behavior. The results are summarized in Table 1, as a comparison with the case without the interference or coherence effects.

Table 1. a) Asymmetry between absorption and emission in Λ and V schemes with maximum interference effects. b) Symmetry between absorption and emission in Λ and V schemes without interference effects.

a)

Interference	Absorption	Emission
Λ	$W_{ab}=0$	$W_e=\frac{\Omega_p^2\gamma}{\Omega^2}\cdot\left[\frac{2\Omega^2}{(\gamma^2/2)+4\Omega^2}\right]\to\frac{1}{2}\frac{\Omega_p^2\gamma}{\Omega^2}$
V	$W_{ab}=\frac{\Omega_p^2\gamma}{\Omega^2}\cdot\left[\frac{\gamma^2}{2\gamma^2+8\Omega^2}\right]\to 0$	$W_e=\frac{\Omega_p^2\gamma}{\Omega^2}$

b)

No Interference	Absorption	Emission
Λ	$\frac{1}{2}\left[\frac{\Omega_p^2\gamma}{(\gamma^2/16)+\Omega^2}\right]$	$\frac{1}{4}\left[\frac{\Omega_p^2\gamma}{(\gamma^2/16)+\Omega^2}\right]$
V	$\frac{1}{4}\left[\frac{\Omega_p^2\gamma}{(\gamma^2/16)+\Omega^2}\right]$	$\frac{1}{2}\left[\frac{\Omega_p^2\gamma}{(\gamma^2/16)+\Omega^2}\right]$

III. V-Type System

For a V-type system the dressed states can be written as, see Fig. 4,

$$|+,n\rangle=\frac{1}{\sqrt{2}}(|c,n\rangle+|b,n+1\rangle),$$
$$|-,n\rangle=\frac{1}{\sqrt{2}}(|c,n\rangle-|b,n+1\rangle). \tag{17}$$

Again we have assumed the resonant driving field. With the same notation as previously applied, we have the equations of motion for the probability amplitudes given by

$$\dot a(t)=-i\Omega_1 b_1(t)-i\Omega_2 b_2(t), \tag{18a}$$
$$\dot b_1(t)=-\Gamma_1 b_1(t)-f b_2(t)-i\Omega_1 a(t), \tag{18b}$$
$$\dot b_2(t)=-\Gamma_2 b_2(t)-f b_1(t)-i\Omega_2 a(t). \tag{18c}$$

The Fano-type interference, as one can see in Fig. 4, is now in the stimulated emission process. For a weak probe field, the transition rate for stimulated emission is now $W_e=-\partial|a(t)|^2/\partial t$, and we have

$$W_e=\frac{\Omega_p^2\gamma}{\Omega^2}. \tag{19}$$

If we consider the case when there are no cross coupling terms ($f=0$) in (18), we have, see (13),

$$W_{e(f=0)}=\frac{\Omega_p^2\gamma}{(\gamma^2/16)+\Omega^2}\,\frac{1}{2}, \tag{20}$$

which is less than the one in (19). In other words, the quantum interference involved in the stimulated emission in V scheme at the two photon resonance is constructive. If we consider a lower-level–driven cascade scheme, it, however, leads to complete cancellation of emission of the cascade scheme like the cancellation of absorption occurs in the Λ scheme.

As in the previous section, for the V-type system the transition rate for absorption is given by

$$W_{ab}=|a(t=\infty)|^2/\tau, \tag{21}$$

where $a(t=0)=0$ and $\tau=\int_0^\infty dt|b_1(t)|^2+|b_2(t)|^2$. Here we have the initial conditions $b_1(0)=-b_2(0)=1/\sqrt{2}$ (since the atomic decay is from $|c\rangle$ to $|b\rangle$, $|b\rangle$ is the initial state for the absorption process). We, then, find the absorption rate as

$$W_{ab}=\frac{\Omega_p^2\gamma}{\Omega^2}\left[\frac{\gamma^2}{2\gamma^2+8\Omega^2}\right]. \tag{22}$$

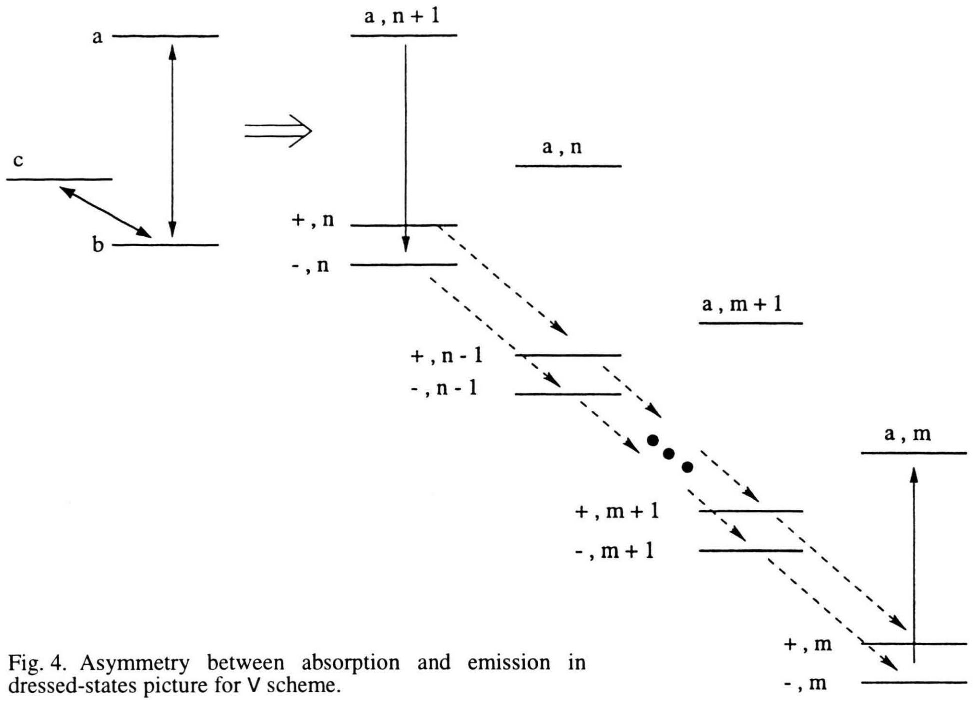

Fig. 4. Asymmetry between absorption and emission in dressed-states picture for V scheme.

If, on the other hand, we assume that the atoms are initially an incoherent superposition of the two lower levels, obviously $W_{ab(\mathrm{incoh})}$ is the same as $W_{e(\mathrm{incoh})}$ for the Λ scheme, such that

$$W_{ab(\,\mathrm{incoh}\,)} = \frac{\gamma\,\Omega_p^2}{(\gamma^2/16) + \Omega^2}\,\frac{1}{4}\,. \tag{23}$$

Comparing (22) with (23), we can see a significant reduction of the absorption, by the factor of the order of $\gamma^2/2\,\Omega^2$, yielding the EIT of V scheme. Here we note that the cancellation of absorption in the V system is qualitatively different from that in the Λ system. The comparisons are summarized in Table 1.

IV. Summary and Conclusions

We present a description of the quantum interference for the absorption and stimulated emission in driven three-level systems. Our discussion has been focused on the resonant situation (for both drive and probe field). The interference effects described here, however, hold for the general two-photon resonance, which will be shown in detail in [17]. The dressed-state description provides a simple physical picture such that the asymmetries between the absorption and the stimulated emission come from the Fano-type interference on one hand, quantum beat-type interference on the other hand. The quantum mechanical two paths are established by the coherent driving field, and the spontaneous emission in the driving transitions plays a crucial role for those asymmetries. The physical origin of the absorption cancellation in the Λ system, in turn, is the reason that leads to the enhancement of stimulated emission in the V system. On the other hand, the *quantum beat*-type interference is responsible for EIT in the V system and also for the emission enhancement in the Λ system.

Acknowledgements

This work was supported by the Office of Naval Research, the Texas Advanced Research and Technology Program, and the Welch Foundation.

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